

DELHI PUBLIC SCHOOL RUBY PARK KOLKATA

REVISION WORKSHEET

CLASS XI (2018-19)

MATHEMATICS

TRIGONOMETRY

1. Simplify the following expression :

$$\frac{\sec(270^\circ - A) \sec(90^\circ - A) - \tan(270^\circ - A) \tan(90^\circ + A)}{\cot A + \tan(180^\circ + A) + \tan(90^\circ + A) + \tan(360^\circ - A) + \cos(180^\circ)}$$

2. Evaluate the value : $3 \left[\sin^4 \left(\frac{3\pi}{2} - \theta \right) + [\sin^4(3\pi + \theta)] \right] - 2 \left[\sin^6 \left(\frac{\pi}{2} + \theta \right) + \sin^6(5\pi - \theta) \right]$

An angle θ is divided into two parts such that the ratio of the tangents of the parts is k ; if the difference between them be φ , then prove that $\sin \varphi = \frac{k-1}{k+1} \sin \theta$

3. If $\cos x + \cos y + \cos z = 0$; $\sin x + \sin y + \sin z = 0$, then find the value of $\cos(x - y) + \cos(y - z) + \cos(z - x)$.

4. If $\frac{a \cos \theta \sec \varphi - x}{a \sin(\theta + \varphi)} = \frac{y - b \sin \theta \sec \varphi}{b \cos(\theta + \varphi)} = \tan \varphi$, evaluate the value of $\frac{x^2}{a^2} + \frac{y^2}{b^2}$.

5. If $\sin \theta = k \sin(\theta + \varphi)$, show that $\tan(\theta + \varphi) = \frac{\sin \varphi}{\cos \varphi - k}$.

6. If $\tan \theta = \frac{x \sin \alpha + y \sin \beta}{x \cos \alpha + y \cos \beta}$ and $x \sin(\theta - \alpha) = k y \sin(\theta - \beta)$, find k .

7. If $\frac{\cot(\alpha - \beta)}{\cot \alpha} + \frac{\cos^2 \gamma}{\cos^2 \alpha} = 1$, then find the value of $\tan^2 \gamma + \tan \alpha \cot \beta$.

8. If $13\theta = \pi$, find the value of $\cos 3\theta + \cos 5\theta + 2 \cos \theta \cos 9\theta$

9. The equation $\sin(\theta + \varphi) = \frac{1}{2} \sin 2\varphi$ is satisfied for two different values of θ , say θ_1 and θ_2 and $\frac{\sin \theta_1 + \sin \theta_2}{\cos \theta_1 + \cos \theta_2} = k \cot \varphi$. Find the value of k .

10. Evaluate : $16 \cos \left(\frac{2\pi}{15} \right) \cos \left(\frac{4\pi}{15} \right) \cos \left(\frac{8\pi}{15} \right) \cos \left(\frac{16\pi}{15} \right)$

11. If $\cot \frac{\theta}{2} = \cot^3 \frac{\varphi}{2}$ and $2 \cot \varphi = \cot \alpha$, prove that $\theta + \varphi = 2\alpha$

12. If in a ΔABC , $\cot A + \cot B + \cot C = 0$, find the value of $\cos A \cos B \cos C$

13. Evaluate : $\tan \alpha + 2 \tan 2\alpha + 4 \tan 4\alpha + 8 \cot 8\alpha - \cot \alpha$.

14. If $\theta = \frac{2\pi}{7}$, then find the value of

(i) $\sec \theta + \sec 2\theta + \sec 4\theta$. (ii) $\cos \theta + \cos 2\theta + \cos 4\theta$ (iii) $\sin \theta + \sin 2\theta + \sin 4\theta$

15. If $\cos \alpha = \cos \beta \cos \varphi = \cos \beta' \cos \varphi'$ and $\sin \alpha = 2 \sin \frac{\varphi}{2} \sin \frac{\varphi'}{2}$ and

$\tan^2 \frac{\alpha}{2} = \lambda \tan^2 \frac{\beta}{2} \tan^2 \frac{\beta'}{2}$, find λ .

16. Prove that $\frac{\cos 7x - \cos 8x}{1 + 2 \cos 5x} = \cos 2x - \cos 3x$.

17. If $\tan \theta \tan \varphi = \sqrt{\frac{a-b}{a+b}}$, then prove that $(a - b \cos 2\theta)(a - b \cos 2\varphi)$ is independent of θ and φ .

18. If $\cos(\theta - \alpha) = a$; $\cos(\theta - \beta) = b$, then evaluate : $\sin^2(\alpha - \beta) + 2ab \cos(\alpha - \beta)$.

19. Find the general solution for the trigonometric equations :

(a) $\sec \theta + 1 = (\sqrt{2} - 1) \tan \theta$

(b) $4 \sin^4 x + \cos^4 x = 1$

(c) $4 \sin \theta \sin 2\theta \sin 4\theta = \sin 3\theta$

(d) $\tan x + \tan 2x + \tan 3x = \tan x \tan 2x \tan 3x$

(e) $4 \cos x + 5 \sin x = 5$, given $\tan 51^\circ 21' = \frac{5}{4}$.

STRAIGHT LINES

- Two consecutive sides of a parallelogram are $4x + 5y = 0$ and $7x + 2y = 0$. If the equation of one diagonal is $11x + 7y = 9$, find the equation of the other diagonal.
- If the straight line $x/a + y/b = 1$ passes through the point of intersection of the lines $x + y = 3$ and $2x - 3y = 1$ and is parallel to $x - y - 6 = 0$, find a and b .
- If the image of the point $(2, 1)$ with respect to a line mirror is $(5, 2)$, find the equation of the mirror.
- The hypotenuse of a right isosceles triangle has its ends at the points $(1, 3)$ and $(-4, 1)$. Find the equations of the legs of the triangle.
- Find the equation of the line mid way between the parallel lines $9x + 6y - 7 = 0$ and $3x + 2y + 6 = 0$.
- Prove that the line $5x - 2y - 1 = 0$ is mid-parallel to the lines $5x - 2y - 9 = 0$ and $5x - 2y + 7 = 0$.
- A vertex of an equilateral triangle is $(2, 3)$ and the opposite side is $x + y = 2$. Find the equations of the other sides.
- If two opposite vertices of a square are $(1, 2)$ and $(5, 8)$, find the coordinates of its other two vertices and the equations of its sides.
- The vertex A of a triangle ABC is given to be $(1, 3)$ and the medians BE and CF are $x - 2y + 1 = 0$ and $y - 1 = 0$. Determine the equations of its sides.
- Find the equation of the line equidistant from the point $(-2, 3)$ and the line $8y = 9x - 12$.
- A line L through $P(1, 2)$ intersect line $L_1: x + 2y + 5 = 0$ at the point A and $L_2: 2x - y + 7 = 0$ at the point B such that P divides AB in the ratio $2:3$ internally. Find the equation of the line L .
- Let a line makes angle 60° with +ve direction of X axis and passes through the point $A(4, 7)$. If it intersects another line $x + y + 2 = 0$ at Q then find length AQ .
- Let a line $y = 2x$ intersects the curve $x^3 - 4y^3 - 3x^2y - 7 = 0$ at A, B and C then find $(OA \cdot OB \cdot OC)$, where O is the origin.
- Find α if the point $(0, \alpha)$ lies within the triangle with sides as $3x + 4y = 12$, $4x + 3y = 12$ and $x - 2y + 10 = 0$.
- A straight line passes through the point $(4, 2)$ and is such that the sum of its intercepts on the coordinate axes is 12 units. Then find the equation of the line.
- Find the coordinate of in-centre of the triangle having vertices $(-36, 7)$, $(20, 7)$, $(0, -8)$.
- The coordinate of a movable point is given by $P[3(\cot \theta + \tan \theta), 4(\cot \theta - \tan \theta)]$, where θ is a variable parameter. Find the locus of the point P .
- The equation of two sides of a square are $3x + 4y - 5 = 0$ and $3x + 4y - 15 = 0$. If $P(6, 5)$ be a point on the third side of the triangle. Find the equation of remaining sides of the triangle.

18. Show that the circles $x^2 + y^2 + 2x - 2y + 1 = 0$ and $x^2 + y^2 - 2x - 2y + 1 = 0$ touch each other externally at $(0, 1)$
19. Find the coordinates of the points on the line $3x + 2y = 5$ which are equidistant from the lines $4x + 3y = 7$ and $2y - 5 = 0$.
20. Find the point to which the origin should be shifted so that the equation $x^2 + y^2 - 5x + 2y - 5 = 0$ will have no first degree term.
21. The three sides of a triangle are given by $(x^2 - y^2)(2x + 3y - 6) = 0$. If the point $(-2, a)$ lies inside and $(b, 1)$ lies outside the triangle, then find the range of a and b .
22. Two sides of a square are along $5x + 12y = 10$ and $5x + 12y + 29 = 0$. The third side passes through $(3, 5)$. Find the other two sides.
23. Show that the distance of the line $ax + by + c = 0$ from the point (x_0, y_0) measured parallel to a line making an angle θ with the positive direction of x -axis is $-\frac{ax_0 + by_0 + c}{a \cos \theta + b \sin \theta}$.
24. Show that the quadrilateral bounded by the lines $y = 0, y = 2, y = \sqrt{3}x$ and $y + 3x = 8\sqrt{3}$ is a cyclic trapezium. Find its vertices and area.
25. The line $3x + 4y = 24$ meets the axes at A and B . The perpendicular bisectors of AB meet the line $y = -1$ at C .
Prove that $\angle ACB$ is a right angle.
26. An equilateral triangle has a side along $5y = 12x - 3$ and the centroid at $(2, -1)$. Find the length of its side.
27. The corners of the base of an isosceles triangle are $A(2a, 0)$ and $B(0, a)$. If one of the equal sides is along $x = 2a$ then find the equations to its other sides and its area.
26. One diagonal of a square is the portion of the line $\sqrt{3}x + y = 2\sqrt{3}$ intercepted by the axes.
Then find the extremities of the other diagonal.

COMPLEX NUMBER

1. If $z = 1 + i \tan\left(\frac{4\pi}{7}\right)$, find $|z|$.
2. **Express** $\frac{1}{1 + \cos \theta + i \sin \theta}$ **in the form** $a + ib$.
3. If $a = \cos \alpha + i \sin \alpha$ and $1 + \sqrt{1 - b^2} = nb$, then find the value of $\frac{b}{2n}(1 + na)\left(1 + \frac{n}{a}\right) - b \cos \alpha$.
4. Convert the complex number $Z = \frac{i-1}{\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}}$ to the polar form.
5. If α and β are two different complex numbers with $|\beta| = 1$, then find $\left| \frac{\beta - \alpha}{1 - \bar{\alpha} \beta} \right|$.
6. If $(x + iy)^3 = u + iv$, then prove that $\frac{u}{x} + \frac{v}{y} = 4(x^2 - y^2)$.
7. Convert the complex number $\frac{-16}{1 + i\sqrt{3}}$ into polar form.
8. If $z = x + iy$ and $w = \frac{1 - iz}{z - 1}$ such that $|z| = 1$, then prove that z is purely real.

DOMAIN AND RANGE

Find the domain and range of the following function

1. $f(x) = \frac{1}{2 - \sin 3x}$
2. $f(x) = \frac{x-2}{3-x}$
3. $f(x) = \sqrt{16 - x^2}$
4. $f(x) = \frac{x^2 - 9}{x - 3}$
5. $f(x) = \frac{x}{1 + x^2}$
6. $f(x) = \frac{1}{\sqrt{x - [x]}}$

SETS

1. In a survey of 400 students in a school , 110 were listed as taking apple juice , 140 taking orange juice and 85 were listed taking both apple as well as orange juice, find how many students were taking neither apple nor orange juice.
2. A college awarded 38 medals in football , 15 in basketball and 20 in cricket. If these medals went to total of 58 men and only 3 men got medals in all 3. How many received medals in exactly 2 of 3 sports?
3. In a group it was found that 21 liked product A , 26 liked product B and 29 liked product C.If 14 people liked products A and B ; 12 liked C and A ; 14 liked B and C .If 8 liked all 3 products , find how many liked only C .
4. The population of a city is 150000 where 3 news papers A b and c are circulated. It has been found 42% read A, 51% read B and 68% read C. Also 30% read A and B , 28% read B and C , 36% read A and C . 8% read none . Find how many read all 3news papers.
5. State and prove De-morgan's laws
6. For two sets A and B if $P(A) = P(B)$ then prove that $A=B$.
7. For two sets B and C , if $A \cup B = A \cup C$; $A \cap B = A \cap C$ for any set A, then prove that $B = C$.