ASSIGNMENT ON RELATION FUNCTION FOR CLASS XII 2018-19:-

1. For real numbers $'x'$ and $'y'$, define xRy, if and only if $x-y+\sqrt{2}$ is an irrational number. Is R transitive? Explain your answer.

2. Let $A=\{a,b,c\}$ and the relation R be defined on A as follows.
   $R=\{(a,a),(b,c),(a,b)\}$.
   Then, write minimum number of ordered pairs to be added in R to make reflexive and transitive.

3. Show that the relation S in set $A=\{x\in\mathbb{Z}:0\le x\le 12\}$ given by $S=\{(a,b):a,b\in A,|a-b|\,is\,divisible\;by\,4\}$ is an equivalence relation. Find the set of all elements related to 1.

4. Let a relation R on the set A of real numbers be defined as $(a,b)\mathbb{R}\Rightarrow 1+ab>0,\forall a,b\in A$. Show that R is reflexive and symmetric but not transitive.

5. Let $f:\mathbb{Z}\to\mathbb{Z}$ be defined by $f(n)=3n$ for all $n\in\mathbb{Z}$ and $g:\mathbb{Z}\to\mathbb{Z}$ be defined by
   $g(n)=\begin{cases}\dfrac{n}{3} & \text{,if } n \text{ is a multiple of 3}\\ 0 & \text{,if } n \text{ is not a multiple of 3}\end{cases}$ for all $n\in\mathbb{Z}$. Find fog and gof.

6. Let $f:\mathbb{R}\to\mathbb{R}$ be a function given by $f(x)=ax+b,\ \forall x\in\mathbb{R}$. Find the constants a and b such that $fof=I_R$.

7. Let $f:A\to A$ be a function such that $fof=f$ show that f is into onto if f is one-one.
   Describe f in this case.

8. Let $f,g:\mathbb{R}\to\mathbb{R}$ be a two function defined as $f(x)=|x|+x$ and $g(x)=|x|-x$ for all $x\in\mathbb{R}$. Then, find fog and gof.

9. Let f and g be real function defined by $f(x)=\dfrac{x}{x+1}$ and $g(x)=\dfrac{x}{x+3}$.Describe the functions gof and fog (if they exist).

10. Let $f:\mathbb{N}\cup\{0\}\to\mathbb{N}\cup\{0\}$ be defined by
    $f(n)=\begin{cases}n+1 & \text{,if } n \text{ is even}\\ n-1 & \text{, if } n \text{ is odd}\end{cases}$ Show that f is invertible and $f=f^{-1}$.

ASSIGNMENT ON CONTINUITY AND DIFFERENTIABILITY FOR CLASS XII 2018-19:-

1. If the function $f(x)=\begin{cases} 3ax+b, & \text{if } x>1 \\ 11, & \text{if } x=1 \\ 5ax-2b, & \text{if } x<1 \end{cases}$ is continuous at $x=1$, then find the values of a and b

2. Test the continuity of the function $f(x)$ at the origin $f(x)=\begin{cases} \dfrac{x}{|x|}, & x\neq 0 \\ 1, & x=0 \end{cases}$

3. For what value of k is the function $f(x)$ continuous at $x=0$

$$f(x)=\begin{cases} \dfrac{\sin x + x\cos x}{x}, & \text{when } x\neq 0 \\ k, & \text{when } x=0 \end{cases}$$

4. Examine the continuity of the following function $f(x)=\begin{cases} \dfrac{x}{2|x|}, & x\neq 0 \\ \dfrac{1}{2}, & x=0 \end{cases}$

5. If $f(x)=\begin{cases} \dfrac{5x+|x|}{3x}, & \text{if } x\neq 0 \\ 2, & \text{if } x=0 \end{cases}$ then show that $f(x)$ is discontinuous at $x=0$.

6. If $f(x)=\begin{cases} x\sin\dfrac{1}{x}, & \text{if } x\neq 0 \\ 0, & \text{if } x=0 \end{cases}$ then discuss the continuity of the $f(x)$ at $x=0$

7. The function $f(x)=\dfrac{\log_e(1+ax)-\log_e(1-bx)}{x}$ is not defined at $x=0$. Find the value of $f(0)$, so that $f(x)$ is continuous at $x=0$

8. Prove that $\lim\limits_{x\to\alpha}\dfrac{x^n g(x)+h(x)}{x^n+1}=\begin{cases} h(x), & \text{when } 0<x<1 \\ \dfrac{1}{2}\{h(x)+g(x)\}, & \text{when } x=1 \\ g(x), & \text{when } x>1 \end{cases}$

9. Evaluate $\lim\limits_{x\to 4}\dfrac{(\cos\alpha)^x-(\sin\alpha)^x-\cos 2\alpha}{x-4}$, $\left(0<\alpha<\dfrac{\pi}{4}\right)$

10. A function $f(x)$ is defined in the following way $f(x)=\begin{cases} -2\sin x, & \text{when } -\pi\le x\le -\dfrac{\pi}{2} \\ a\sin x+b, & \text{when } -\dfrac{\pi}{2}<x<\dfrac{\pi}{2} \\ \cos x, & \text{when } \dfrac{\pi}{2}\le x\le \pi \end{cases}$

If $f(x)$ is continuous in $-\pi \le x \le \pi$, then find a and b.

11. Find the values of a and b such that the function $f(x)=\begin{cases} x+a\sqrt{2}\sin x, & \text{when } 0\le x<\frac{\pi}{4} \\ 2x\cot x+b, & \text{when } \frac{\pi}{4}\le x\le\frac{\pi}{2} \\ a\cos 2x-b\sin x, & \text{when } \frac{\pi}{2}<x\le\pi \end{cases}$

is continuous for $0\le x\le\pi.$

12. Show that the function $\lim\limits_{n\to\infty}\dfrac{\cos\pi x-x^{2n}\sin(x-1)}{1+x^{2n+1}-x^{2n}}$ is continuous at $x=1.$

13. The value of $f(0)$, so that the function $f(x)=\sqrt{a^2-ax+x^2}-f(x)=\sqrt{a^2+ax+x^2}$ becomes continuous for all x is given by $f(0)=k,$ then find $k.$

14. Show that the function $f(x)=\begin{cases} |x-a|\sin\dfrac{1}{x-a}, & if\ \ x\ne a \\ 0, & if\ \ x=a \end{cases}$ is continuous at $x=a$.

15. For what value of $'k'$, the function $f(x)=\begin{cases} \dfrac{\sin 2x}{x}, & x\ne 0 \\ k, & x=0 \end{cases}$ is continuous at $x=0$?

16. The value of $f(0)$, so that the function $f(x)=\dfrac{(27-2x)^{\frac{1}{3}}-3}{9-3(243+5x)^{\frac{1}{5}}}\ (x\ne 0)$ becomes continuous for all x is given by $f(0)=k,$ then find $k.$

17. The value of $f(0)$, so that the function $f(x)=\dfrac{2-(256-7x)^{\frac{1}{8}}}{(5x+32)^{\frac{1}{5}}-2}\ (x\ne 0)$ becomes continuous for all x is given by $f(0)=k,$ then find $k.$

18. Find the value of the constants a,b and c for which the function $f(x)=\begin{cases} (1+ax)^{\frac{1}{x}}, & x<0 \\ b, & x=0 \\ \dfrac{(x+c)^{\frac{1}{3}}-1}{(x+1)^{\frac{1}{2}}-1}, & x>0 \end{cases}$

may be continuous at $x=0$

19. Test the differentiability of $f(x)=|\sin x-\cos x|$ at $x=\frac{\pi}{4}$.

20. If $f(x)=\begin{cases} ax^2+1, & \text{if } |x|<1 \\ \dfrac{1}{|x|}, & \text{if } |x\ge 1| \end{cases}$ is differentiable at $x=1$, then find a and $b.$

ASSIGNMENT ON INVERSE FOR CLASS XII (2018-19):-

1. Simplify : $\cot^{-1}\left(\dfrac{\sqrt{1+\sin x}+\sqrt{1+\sin x}}{\sqrt{1+\sin x}-\sqrt{1+\sin x}}\right);\ x\in\left(\dfrac{\pi}{2},\pi\right)$

2. Find the value of $\sin\left(\dfrac{1}{4}\sin^{-1}\dfrac{\sqrt{63}}{8}\right)$.

3. Solve for $x:\ \sin\left[2\cos^{-1}\left\{\cot\left(2\tan^{-1}x\right)\right\}\right]=0.$

4. If $\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\pi,$ then prove that $x^4+y^4+z^4+4x^2y^2z^2=2\left(x^2y^2+y^2z^2+z^2x^2\right).$

5. Solve for $x:\ \cos^{-1}\dfrac{x^2-1}{x^2+1}+\tan^{-1}\dfrac{2x}{x^2-1}=\dfrac{2\pi}{3}.$

6. If $\left(\sin^{-1}x\right)^2+\left(\sin^{-1}x\right)^2+\left(\sin^{-1}x\right)^2=\dfrac{3\pi^2}{4},$ then find the minimum value of $x+y+z.$

7. Find the values of x for which $\sin^{-1}\left(\cos^{-1}x\right)<1$ and $\cos^{-1}\left(\cos^{-1}x\right)<1.$

8. Solve the equation $\sec^{-1}\dfrac{x}{a}-\sec^{-1}\dfrac{x}{b}=\sec^{-1}b-\sec^{-1}a, a\geq 1,\ b\geq 1,\ a\neq b.$

9. Solve for real values of $x:\dfrac{\left(\sin^{-1}x\right)^3+\left(\cos^{-1}x\right)^3}{\left(\tan^{-1}x+\cot^{-1}x\right)^3}=7$.

10. Let $f\left(x\right)=\sin x+\cos x+\tan x+\sin^{-1}x+\cos^{-1}x+\tan^{-1}x.$ Then find the minimum and maximum value of $f\left(x\right).$

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